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ON THE EFFECTS OF VISCOUS DISSIPATION AND PRESSURE WORK IN FREE CONVECTION LOOPS

HAIM H. BAU
Department of Mechanical Engineering and Applied Mechanics, University of Pennsylvania, Philadelphia, PA 19104, U.S.A.

and

K. E. TORRANCE
Sibley School of Mechanical and Aerospace Engineering, Cornell University, Ithaca, NY 14853, U.S.A.

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Abstract The effects of viscous dissipation and pressure work are examined theoretically for laminar free convection loops. The appropriate governing equations are derived. Whereas previous work has considered only dissipation effects, the present paper shows that dissipation and pressure work effects are of comparable magnitude and must be considered together. Analytical solutions are presented for several open and closed loops. Both constant flux and constant temperature heating conditions are examined. Viscous dissipation and pressure work effects are found to have opposing influences on the flow in a loop. The former can enhance a flow for certain heating orientations, but the latter is usually dominant and retards a flow.

NOMENCLATURE

- b, vertical dimension of the loop;
- B, inertia parameter, $\beta g\Delta T^*/bF^2$;
- c_p, specific heat;
- d, diameter of the loop cross section;
- D, dissipation parameter, $\beta gb/c_p$;
- F, friction coefficient (i.e. for Poiseuille flow $32\nu/d^2$);
- g, acceleration of gravity;
- H, heat addition at the loop wall expressed per unit length and per unit internal cross sectional area;
- h, heat transfer coefficient;
- k, dimensionless heat transfer coefficient, $4Fhb/d\rho_0^*c_p\beta g\Delta T^*$;
- L, total length of the loop;
- P, pressure;
- p, pressure perturbation from adiabatic state;
- q, heat flux per unit area of loop wall;
- s, coordinate along the loop;
- t, time;
- T, temperature;
- ΔT, temperature difference characteristic of heating and cooling conditions;
- u, velocity.

Greek symbols

- β, thermal expansion coefficient;
- γ, Gruneisen parameter, $\beta/\rho_0^*c_p\chi$;
- δ, length scale appropriate for viscous effects;
- θ, temperature perturbation from adiabatic state;
- μ, dynamic viscosity;
- ν, kinematic viscosity;
- ρ, density;
- ϕ, proportionality parameter in $\Phi = \phi u^2$;
- Φ, dissipation function (i.e. for Poiseuille flow $32\mu u^{*2}/d^2$);
- ω, cosine of angle between vertical and local flow direction;
- χ, isothermal compressibility.

Special symbol

- D/Dt, substantial derivative, $\partial/\partial t + u\partial/\partial s$.

Superscript

- $*$, dimensional quantity.

Subscript

- a, adiabatic rest state;
- 0, reference condition.

1. INTRODUCTION

IN A FREE convection loop, fluid is circulated by simply heating or cooling various sections of the loop. Free convection loops arise in many geophysical and technical applications [1, 2]. The prediction of fluid circulation rates is a topic of considerable interest. The present paper examines the effects of both viscous dissipation and pressure work on the performance of free convection loops.

Prior studies of free convection loops have concentrated on the starting transients, the steady state motion, and the stability of the steady state motion. Welander [3] analyzed the stability of a closed loop consisting of two vertical, adiabatic legs. A point heat source connected the legs at the bottom, and a point heat sink connected the legs at the top. Creveling *et al.* [4] considered a toroidal loop and carried out analyses and confirming experiments. Bau and Torrance [5] examined the start up and stability of an open loop. The foregoing analyses employed 1-dim. heat balances for the fluid in the loop. The rate of change of convected enthalpy was balanced by the rate of heating or cooling at the loop walls. Viscous dissipation and pressure work were not included.

Several studies have considered the influence of stress work on free convection flows. Viscous dissipation (without pressure work) was considered in the natural convection boundary layer adjacent to a heated vertical plate by Gebhart [6] and Gebhart and Mollendorf [7]. Pressure work was considered by Kuiken [8]. For this external natural convection flow, pressure work is always more important than viscous dissipation for gases [8, 9] and at least as important for liquids [9]. Thus, when viscous dissipation is considered, pressure work must be included. Similar results have been obtained for natural convection flows in general [10], and for Bénard convection in particular [11].

The foregoing studies show that viscous dissipation and pressure work effects in free convection flows are proportional to a dissipation parameter, defined by $D = \beta g b/c_p$. The dissipation parameter expresses the ratio of the vertical length scale for a natural convection flow, b, to the vertical length scale over which stress work effects are important, $c_p/\beta g$.

In a recent paper, Zvirin [12] extended the analyses of Welander [3] and Creveling *et al.* [4] for free convection loops to include viscous dissipation. Pressure work was neglected. Consequently, the energy equation yields a net production of sensible energy by viscous dissipation which is in violation of the first law of thermodynamics.

In this paper, transport equations are derived which are appropriate for free convection loops when viscous dissipation and pressure work are present. Analytical solutions are obtained and discussed for several loop geometries.

2. FORMULATION

The basic 1-dim. equations for the conservation of mass, momentum and energy [13] for flow in a loop are given by

$$\frac{D\rho^*}{Dt^*} + \rho^* \frac{\partial u^*}{\partial s^*} = 0, \tag{1}$$

$$\frac{Du^*}{Dt^*} + Fu^* + g\omega = -\frac{1}{\rho^*}\frac{\partial P^*}{\partial s^*}, \tag{2}$$

$$\rho^* c_p \frac{DT^*}{Dt^*} - \beta T^* \frac{DP^*}{Dt^*} = \Phi^* + H^*. \tag{3}$$

All variables are averaged over the flow cross section and are defined in the Nomenclature. The asterisks denote dimensional quantities. The same variables without asterisks will later be nondimensional. The state equation is expressed in the linearized form

$$\frac{\rho^* - \rho_0^*}{\rho_0^*} = -\beta(T^* - T_0^*) + \chi(P^* - P_0^*) \tag{4}$$

where the subscript 0 denotes a reference condition. The foregoing equations will be applied to a loop of length L and vertical height b.

The momentum equation (2) expresses a balance between inertia, friction, buoyancy, and pressure forces. The parameter F represents the loop friction per unit length. For Poiseuille flow in a circular tube of diameter d, $F = 32\nu/d^2$. For turbulent flow, F depends on u^*. The present analysis is restricted to laminar flow. In the body force term, ω denotes the cosine of the angle between the vertical and the local flow direction (or direction of s). In the energy equation (3), axial heat conduction has been neglected. The term Φ^* is the viscous dissipation function. For Poiseuille flow in a circular tube, $\Phi^* = 32\mu u^{*2}/d^2$. The term H^* is the heat addition at the tube wall expressed per unit length and per unit internal cross sectional area. For example, if a heat flux q^* is supplied per unit area of the tube wall, then $H^* = 4q^*/d$.

In the absence of convection and heat addition an adiabatic rest state (denoted by the subscript a) can be defined in the following manner:

$$\frac{dP_a^*}{ds^*} = -\rho_a^* g\omega, \quad \frac{dT_a^*}{ds^*} = -\frac{\beta g T_a^*}{c_p}\omega,$$
$$\frac{\rho_a^* - \rho_0^*}{\rho_0^*} = -\beta(T_a^* - T_0^*) + \chi(P_a^* - P_0^*). \tag{5}$$

The actual state will be considered as a perturbation of this base state.

Scaling quantities may be obtained with heuristic reasoning. Let the total variation of temperature along the loop be given by ΔT^*. Then

$$T^* - T_a^* \sim \Delta T^*.$$

From equations (2) and (4), it follows that

$$u^* \sim \beta g \Delta T^*/F$$

and

$$P^* - P_a^* \sim \rho_0^* \beta g \Delta T^* b.$$

In steady state free convection flows the buoyancy is balanced by a viscous force, hence

$$\mu u^*/\delta^2 \sim \rho_0^* \beta g \Delta T^*$$

where δ is a length scale appropriate to viscous effects. The viscous dissipation (Φ^*) is proportional to the viscosity and the square of the velocity gradient. Thus

$$\Phi^* \sim \mu(u^*/\delta)^2 \sim \rho_0^*(g\beta\Delta T^*)^2/F.$$

Later, the symbol ΔT^* will be used to denote a temperature difference characteristic of the imposed heating and cooling conditions. Therefore, a set of nondimensional quantities may be defined in the following manner:

$$u = \frac{F}{\beta g \Delta T^*}u^*, \quad \theta = \frac{T^* - T_a^*}{\Delta T^*}, \quad s = \frac{s^*}{b},$$

$$t = \frac{\beta g \Delta T^*}{bF}t^*,$$

$$p = \frac{P^* - P_a^*}{\rho_0^* \beta g \Delta T^* b}, \quad \Phi = \frac{F}{\rho_0^*(\beta g \Delta T^*)^2}\Phi^*,$$

$$q = \frac{bF}{\beta g \rho_0^* c_p (\Delta T^*)^2} H^*. \quad (6)$$

Subtracting the adiabatic state equation in equation (5) from equation (4), a state equation is obtained in which nondimensional parameters appear,

$$\frac{\rho^* - \rho_a^*}{\rho_0^*} = (\beta \Delta T^*)\left(-\theta + \frac{D}{\gamma} p\right). \quad (7)$$

In this expression, $D = \beta g b / c_p$ is the dissipation parameter and $\gamma = \beta / \rho_0^* c_p \chi$ is known as the Gruneisen parameter [14]. By integrating equation (5), it may be shown that the adiabatic density satisfies

$$\frac{\rho_a^*}{\rho_0^*} = \mathrm{e}^{\,D\omega s/\gamma} + \frac{\gamma \beta T_0^*}{1-\gamma}(\mathrm{e}^{-D\omega s} - \mathrm{e}^{-D\omega s/\gamma}). \quad (8)$$

In most cases of practical interest $\gamma = O(1)$ and $D \ll 1$. Consequently one may neglect the pressure dependence in equation (7) and may assume that $\rho_a^* \sim \rho_0^*$ in equation (8). Employing equation (6), and expanding the adiabatic rest state, the following are obtained from equations (1)–(3):

$$\frac{\partial u}{\partial s} = 0, \quad (9)$$

$$B\frac{\partial u}{\partial t} + u - \theta\omega = -\frac{\partial p}{\partial s}, \quad (10)$$

$$\frac{\mathrm{D}\theta}{\mathrm{D}t} + Du\theta[1 + (\beta T_a^*)]\omega - D(\beta T_a^*)\frac{\mathrm{D}p}{\mathrm{D}t} = D\phi u^2 + q \quad (11)$$

where $B = \beta g \Delta T^* / bF^2$ is an inertial parameter and quantities of $O(\beta \Delta T^*)$ and higher order terms in D have been neglected. We have assumed that the viscous dissipation function (Φ) is proportional to the square of the mean velocity ($\Phi = \phi u^2$). This is exactly true for Poiseuille flow, where in the present notation $\phi \equiv 1$.

Equations (9)–(11) differ from the usual Boussinesq equations applied in loop calculations [3–5] by the presence of the three additional terms involving D in the energy equation. The first two terms respectively represent hydrostatic and dynamic pressure work. The third term is due to viscous dissipation. We emphasize that all three terms are of the same order in D. In the limit $D \to 0$, equation (11) approaches the familiar 1-dim. energy equation used in refs. [3–5]. For finite values of D, all three terms must be retained to treat dissipation effects correctly.

Typically, $\beta T_a^* \sim 1$ for gases and $\beta T_a^* \ll 1$ for liquids. The present analysis is restricted to liquids; hence, the terms containing the group βT_a^* can be neglected. The analysis is also restricted to closed loops or to open loops with no pressure difference between inlet and outlet. Applying the above assumptions and integrating equation (10) around a loop, one obtains,

$$u = u(t), \quad (12)$$

$$B\frac{\partial u}{\partial t} + u = \frac{1}{(L/b)}\oint \theta\omega \,\mathrm{d}s, \quad (13)$$

$$\frac{\mathrm{D}\theta}{\mathrm{D}t} + Du\theta\omega = D\phi u^2 + q. \quad (14)$$

Under steady state conditions equations (12)–(14) become

$$u = \frac{1}{(L/b)}\oint \theta\omega \,\mathrm{d}s, \quad (15)$$

$$u\frac{\partial \theta}{\partial s} + Du\theta\omega = D\phi u^2 + q. \quad (16)$$

For the case of a closed loop, the temperature is continuous. Integration of the energy equation (16) around the loop yields

$$Du\oint \theta\omega \,\mathrm{d}s = D\phi(L/b)u^2 + \oint q \,\mathrm{d}s. \quad (17)$$

The last term on the RHS is zero since the heat added to the loop must equal the heat removed. The result is an integral equality between the dissipation term and the remaining pressure work term. That is, the two effects must balance at steady state. Note also that this equality is a kinetic energy equation. After dividing by u and comparing with the momentum equation (15), we see that $\phi \equiv 1$. That is, ϕ is not an independent parameter as assumed by Zvirin [12]. Similar results can be obtained for an open loop.

The foregoing result has some important consequences. Although it appears at first that equations (15) and (16) represent a coupled set for u and $\theta(s)$, in fact, equation (15) can be recovered from equation (16). Thus, it is sufficient to obtain a solution of the energy equation (16).

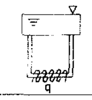
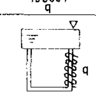
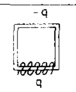
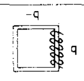
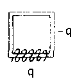
3. ILLUSTRATIVE SOLUTIONS

Steady state flows in free convection loops are considered in this section. Specific attention is given to viscous dissipation and pressure work effects which are expressed in terms of the dissipation parameter D. Seven different loop geometries are examined (Tables 1 and 2). Results are presented in terms of flow rates (Figs. 1–3) and temperature distributions (Figs. 4 and 5) in the loops.

Two types of loops are examined. Loops I, II and VI (Tables 1 and 2) are open loops which are each connected at the top to an isothermal reservoir. The inlet and outlet pressures are equal. Loops III, IV, V and VII are closed loops. All loops lie in a vertical plane and consist of connected sequences of straight pipes. The pipe segments are of unit length (dimensional length b). Fluid circulates in a counterclockwise sense.

The loops illustrated in Table 1 have constant flux heaters in the bottom leg or in the ascending leg. Positive values of q^* (dimensional) or q (nondimensional) denote heat addition. In the open loops, heat removal is accomplished by the reservoirs. The reservoirs are at the adiabatic fluid temperature, $T_a^*(b)$, appropriate for the top of the loop. In the closed loops,

Table 1. Fluid velocities in loops with constant flux heating and cooling

LOOP	GEOMETRY	STEADY STATE VELOCITY SQUARED, u^2		
		$u^2(D)$	$D \to 0$	$D \to \infty$
I		$q\dfrac{1-e^{-D}}{2-2e^{-D}+De^{-D}}$	$\frac{q}{3}(1+\frac{D}{6})$	$\frac{q}{2}$
II		$q\dfrac{D-1+e^{-D}}{D(2-2e^{-D}+De^{-D})}$	$\frac{q}{6}(1+\frac{D}{3})$	$\frac{q}{2}$
III		$q\dfrac{1-e^{-D}}{2(1-e^{-D})+D(1+e^{-D})}$	$\frac{q}{4}(1-\frac{7}{24}D^2)$	$\frac{q}{D}$
IV		$\dfrac{q}{2D}\dfrac{D-1+e^{-D}}{1-e^{-D}+\frac{D}{2}(1+e^{-D})}$	$\frac{q}{8}(1+\frac{D}{6})$	$\frac{q}{D}$
V		$\dfrac{q}{D}\dfrac{1-De^{-D}-e^{-D}}{2(1-e^{-D})+D(1-e^{-D})}$	$\frac{q}{8}(1-\frac{D}{6})$	$\frac{q}{D^2}$

heat removal is accomplished at a rate $-q^*$ by a constant flux process. This is assumed for simplicity but is not, of course, generally practical. The remaining loop segments are adiabatic.

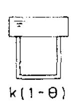
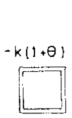
The loops shown in Table 2 differ in that they have constant temperature heaters and coolers. Heat transfer to the circulating fluid is assumed to be given by $q^* = h(T_w^* - T_f^*)$ where h is a heat transfer coefficient, T_w^* is the tube wall temperature (assumed constant), and T_f^* is the local mean fluid temperature. The heater wall temperature is $T_a^*(0)+\Delta T^*$ and the cooler wall temperature (in loop VII) is $T_a^*(b)-\Delta T^*$. The first term in these expressions is the local adiabatic temperature and the second term is the imposed temperature difference relative to adiabatic. The dimensionless heat flux becomes

$$\left.\begin{aligned} q &= k(1-\theta) \quad \text{in a heater} \\ q &= -k(1+\theta) \quad \text{in a cooler} \end{aligned}\right\} \tag{18}$$

where $k = 4Fhb/d\rho_0^* c_p \beta g \Delta T^*$ is a dimensionless heat transfer coefficient.

The loops shown in Tables 1 and 2 allow various heating and cooling arrangements to be studied and compared. The simple geometries lead to closed form analytical solutions of equations (15) and (16). Solutions are tabulated in Tables 1 and 2. The tabulations include the general solution (third column) and first order approximations in the limits of $D \ll 1$ (fourth column) and $D \to \infty$ (fifth column). The limit $D \to \infty$ must be regarded as purely illustrative since it is beyond the limit of applicability of the approximate governing equations. In Table 1, results are given for the square of the steady state loop velocity, u^2. In Table 2, solutions are given in terms of u, which appears implicitly with k and D as parameters. In the columns labeled $D \to 0$ in Tables 1 and 2, the coefficient of the term in parentheses is the steady state solution in the absence of stress work effects (i.e. $D = 0$).

Table 2. Fluid velocities in loops with constant temperature heating and cooling

LOOP	GEOMETRY	STEADY STATE VELOCITY, u		
		$u(D)$	$D \to 0$	$D \to \infty$
VI		$u=\dfrac{(1-e^{-k/u})(1+\frac{Du^2}{k})(1-e^{-D})}{D+(2-e^{D}-e^{-D})e^{-k/u}+(e^{D}-e^{-D})}$	$u=\dfrac{1-e^{-k/u}}{3}\left(1+\dfrac{Du^2}{k}-\dfrac{D}{2}+\dfrac{De^{-k/u}}{3}\right)$	$u=e^{-D}$
VII		$u=\dfrac{1-e^{-k/u}}{1+e^{-k/u}} \times \dfrac{(e^{D}-e^{-D})+\frac{Du^2}{k}(2-e^{D}-e^{-D})}{2D+e^{D}-e^{-D}}$	$u=\dfrac{1}{2}\dfrac{1-e^{-k/u}}{1+e^{-k/u}}\left(1+\dfrac{D^2}{12}-\dfrac{D^2u^2}{2k}\right)$	$u=\dfrac{1-e^{-k/u}}{1+e^{-k/u}}\left(1-\dfrac{Du^2}{k}\right)$

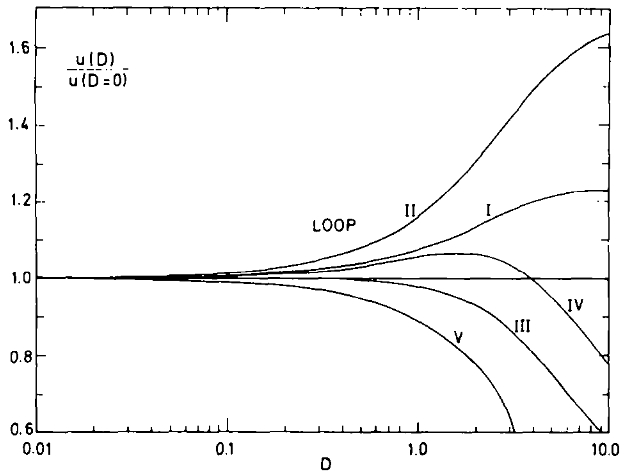


FIG. 1. Ratio of fluid velocity with dissipation to fluid velocity without dissipation for loops with constant flux heating and cooling. Loops I and II are open loops; the others are closed loops.

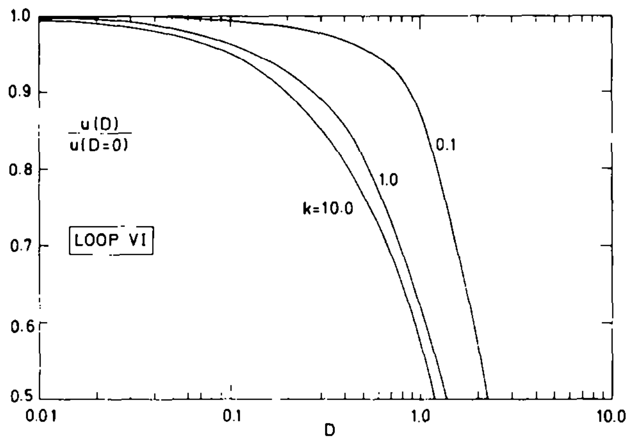


FIG. 2. Ratio of fluid velocity with dissipation to fluid velocity without dissipation for open loop VI. Constant temperature heating; k is a dimensionless internal heat transfer coefficient.

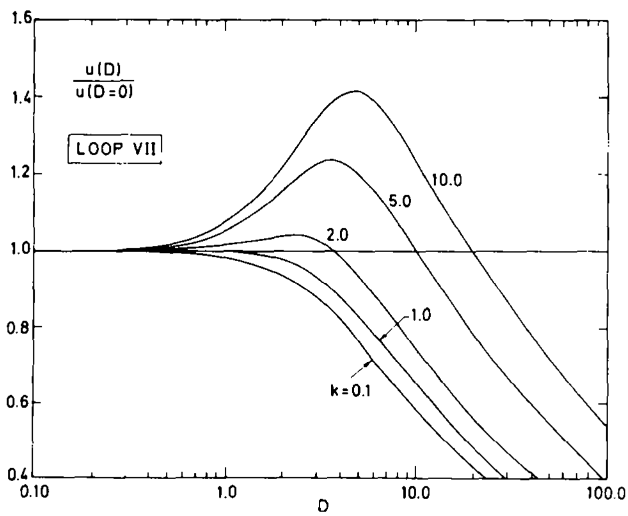


FIG. 3. Ratio of fluid velocity with dissipation to fluid velocity without dissipation for closed loop VII. Constant temperature heating and cooling; k is a dimensionless internal heat transfer coefficient.

In the limit of no dissipation, $D = 0$, there are several points to be observed. For constant flux heating in Table 1, the largest velocities in a given type of loop occur with bottom heating and top cooling (loops I and III). Smaller velocities are obtained when heating or cooling is applied to a vertical leg (loops II, IV and V); results for loops IV and V are identical.

For $D = 0$ and a given geometry of heat addition, values of u^2 in the closed and open loops are in the ratio 3 : 4. This is the inverse ratio of the piping resistance in the two loops. The foregoing observations are generally similar to the trends for $D = 0$ and constant temperature heating in Table 2.

The analytical solutions for intermediate values of D are graphed in Fig. 1 for constant flux heating and in Figs. 2 and 3 for constant temperature heating. In these figures, the ordinate is the fluid velocity normalized by the velocity without dissipation ($D = 0$). Clearly a wide spectrum of behavior is apparent. To understand this behavior, it will be helpful to examine in detail the buoyancy generation processes within the loops. Buoyancy is, of course, influenced by viscous dissipation, pressure work, and the heating and cooling arrangement. Open loops are considered first, followed by closed loops.

Representative temperature distributions are shown in Fig. 4 for an open loop (loop I) with constant flux heating. The abscissa is the distance coordinate measured counterclockwise from the upper left corner of the loop. The ordinate is the fluid temperature expressed as $\theta(s)/q^{1/2}$. The inlet temperature is the reservoir temperature $\theta = 0$. The temperature distributions may be interpreted with the aid of the fluid energy equation (16). For $D = 0$, the fluid temperature increases linearly across the heater while the vertical legs are isothermal. The buoyant drive is found by integrating the temperature difference between the vertical legs over the height of the loop [equation (15)].

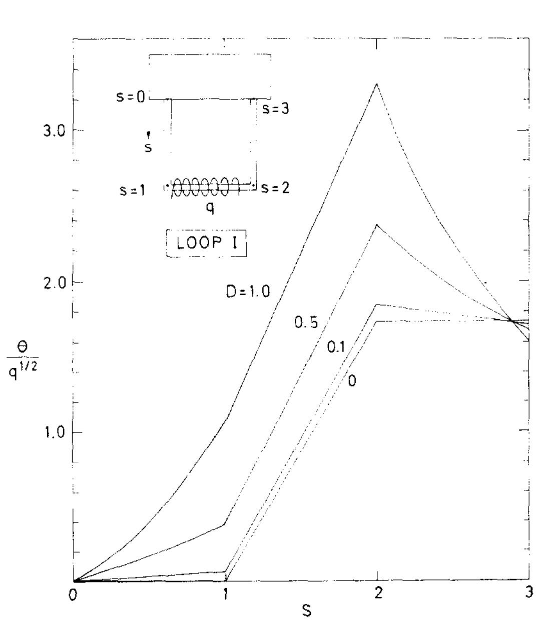


FIG. 4. Temperature distribution around loop I for various values of the dissipation parameter D.

For $D > 0$, viscous dissipation and pressure work both become important. Viscous dissipation causes the fluid temperature to increase linearly with s. This may be seen by integrating the first and third terms in equation (16) to obtain the component due to viscous dissipation: $\theta = Dus$. Thus, viscous heating always enhances the buoyant drive in an open loop and leads to an increase of the fluid velocity as seen in Fig. 1 for loops I and II. Other values of $u(D)/u(D = 0)$ above unity in Figs. 1–3 can be attributed to this effect.

The pressure work term, on the other hand, depends on the magnitude of the temperature θ and always leads to an exponential *increase* of fluid temperature in the descending leg [the term $Du\theta\omega$ in equation (16) is negative] and to an exponential *decrease* of temperature in the ascending leg ($Du\theta\omega$ is positive). As a result of pressure work, descending fluid tends to get warmer and ascending fluid cooler. This reduces the buoyant drive and leads to curvature in the temperature profiles in Fig. 4 in the regions $0 \leqslant s \leqslant 1$ and $2 \leqslant s \leqslant 3$. Pressure work has no influence in the horizontal leg (or legs). The actual reduction of the buoyant drive varies from loop to loop, since the temperature θ appearing in the pressure work term is a function of the heating and cooling arrangements in each loop. Nevertheless, the pressure work term tends to counter the buoyancy production by viscous dissipation (which increases with D). This leads to the decay and asymptotic behavior of $u(D)/u(D = 0)$ at large D in Figs. 1 and 3. The decay at large D in Fig. 2 is a result of both pressure work and reduced heat transfer, as we shall see next.

For the case of an open loop with a constant temperature heater, as in loop VI in Table 2, viscous dissipation and pressure work effects also oppose one another. However, in this loop an additional factor arises which further reduces the buoyant drive. Viscous heating increases the fluid temperature at the inlet to the heater. Since the heater effectiveness depends on the fluid-to-wall temperature difference, the heat transfer is reduced. In turn, this reduces the buoyant drive and leads to the decay of the velocity in Fig. 2. This contrasts with the asymptotic values of $(3/2)^{1/2}$ and $3^{1/2}$ for loops I and II in Fig. 1 as $D \to \infty$.

The behavior of closed loops is considered next. Closed loop velocities are given in Fig. 1 for constant flux heating (loops III, IV, and V) and in Fig. 5 for constant temperature heating (loop VII). Representative temperature distributions for loop III are shown in Fig. 5. Without loss of generality, the loop temperatures at $s = 0$ have been scaled to $\theta = 0$. The temperature distributions from $s = 0$ to $s = 3$ are quite similar to the open loop results in Fig. 4. In contrast with the open loop, however, the temperature in the closed loop changes smoothly from $s = 3$–4 as a result of the external cooling. Scaled temperatures in the closed loop are generally slightly higher than in the open loop. This is a consequence of the lower fluid

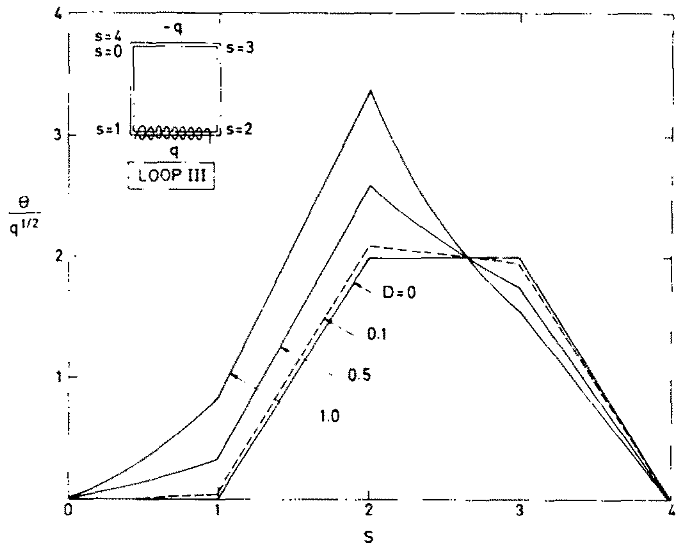


FIG. 5. Temperature distributions around loop III for various values of the dissipation parameter D.

velocities caused by the added frictional resistance of the upper segment. Stress work effects in a closed loop are similar to those in an open loop. That is, the effects of viscous heating are countered by the effects of pressure work. In a closed loop, viscous dissipation is greater by the heating produced in the upper leg. In addition, since the fluid circulates continuously, dissipation leads to an increase in the average temperature of the loop. In turn, higher local temperatures tend to increase the magnitude of the pressure work effect.

Flow rates in the closed loops are due to buoyancy production by the heating arrangement, dissipation, and pressure work. Velocities in loop III in Fig. 1 are damped primarily by the temperature-enhanced pressure work effect. Moving the heater to the ascending leg, as in loop IV in Fig. 1, lowers the mean temperature in the leg and thus the influence of pressure work. Consequently, velocities in loop IV are enhanced over an intermediate range of D. Moving the cooler to the ascending leg, as in loop V, means that the leg temperature remains high only by virtue of the upstream heater. Pressure work effectively lowers the flow velocity. As $D \to \infty$, the velocities of all three closed loops in Fig. 1 approach zero.

Results for a closed loop with constant temperature heaters are shown in Fig. 3. Again, stress work effects may accelerate or decelerate the flow, and the effect depends on the heat transfer coefficient k. In all cases, the flow slows down for large values of D.

4. DISCUSSION

The previous section describes laminar steady state flows in several free convection loops when stress work effects are important. The results indicate that whenever viscous dissipation is included, pressure work must also be included. The stress work effects depend on the magnitude of the dissipation parameter D.

The buoyant drive in a loop is influenced by viscous dissipation, pressure work, and the arrangement and type of heaters and coolers. In general, viscous dissipation tends to increase the buoyant drive. This is countered by pressure work, which always tends to reduce the buoyant drive. The two effects compete. When dissipation dominates the loop velocity can be increased. This occurs over limited parameter ranges. When pressure work dominates, as it usually does, the loop velocity is retarded. The two stress work effects depend on the heating arrangements and can yield a wide variety of flow behaviors as D is increased.

Certain special cases may be noted. Viscous dissipation in an open loop always enhances the buoyant drive. In constant temperature heaters, it can interfere with the heat transfer and slow down the flow. In closed loops, dissipation tends to increase the mean temperature of the loop. The pressure work effect depends on the local fluid temperature. Thus, this process is strongly influenced by dissipation and by the location of heaters and coolers. Flow rates can change dramatically when heaters and coolers are moved. The temperature-dependence of the pressure work is often the dominant influence on the resulting flows.

Stress work effects are important in free convection loops only under very exceptional circumstances. Values of the dissipation parameter $D = \beta g b/c_p$ for liquids under typical ambient conditions and normal gravity, and with a vertical length scale of $b = 1$ m, are: water, 0.4×10^{-6}; glycerin, 2×10^{-6}; mercury, 13×10^{-6}; and carbon dioxide, 75×10^{-6}. From our results, D must be greater than 10^{-2} to cause a 1% change in loop velocities. Such values are achieved only in strong gravity fields or for length scales of geophysical dimensions.

The present work highlights some interesting facts. The energy equation (16) really contains two balances. When integrated around a free convection loop, one balance says that the sensible heat added must equal the sensible heat removed. For a closed loop the implications are obvious. From equation (17), there results

$$\oint q \, ds = 0.$$

For an open loop, the heat discharged must equal the heat added. Thus, in Fig. 4, the loop discharge temperature decreases as u increases since the product $u\theta$ at exit is a constant. The second balance obtained from integrating equation (16) around a loop is

$$Du \oint \theta\omega \, ds = D(L/b)u^2. \quad (19)$$

This states that the pressure work done on the fluid must exactly balance the heat produced by viscous dissipation. Failure to include the pressure work term can lead to an erroneous buoyancy production and to a net production of energy when viscous dissipation is added to a natural convection flow [6, 7, 12].

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EFFETS DE LA DISSIPATION VISQUEUSE ET DU TRAVAIL DE PRESSION DANS LES BOUCLES A CONVECTION NATURELLE

Résumé On examine théoriquement les effets de la dissipation visqueuse et du travail de pression dans les boucles de convection naturelle laminaire. On établit les équations de base. Alors que les travaux antérieurs n'ont considérés que les effets de la dissipation, le texte présent montre que les effets de la dissipation et du travail de pression sont d'ordre de grandeur comparable et doivent être considérés ensemble. Des solutions analytiques sont présentées pour quelques boucles ouvertes ou fermées. On examine des conditions de chauffage à flux constants et à température constante. La dissipation visqueuse et le travail de pression ont des effets d'influences opposées sur l'écoulement dans la boucle. Le dernier peut favoriser l'écoulement pour certaines orientations de chauffage, mais le premier est usuellement dominant et il retarde l'écoulement.

EINFLUSS DER VISKOSEN DISSIPATION UND DER DRUCKARBEIT IN KREISLÄUFEN MIT FREIER KONVEKTION

Zusammenfassung – Es wurde der Einfluß der viskosen Dissipation und der Druckarbeit in Kreisläufen mit laminarer Strömung und freier Konvektion theoretisch untersucht. Die zugehörigen erforderlichen Bestimmungsgleichungen werden hergeleitet. Im Gegensatz zu früheren Arbeiten, welche nur Dissipationseffekte berücksichtigten, zeigt die vorliegende Arbeit, daß der Einfluß der Dissipation und der Druckarbeit von gleicher Größenordnung sind und zusammen betrachtet werden müssen. Für einige offene und geschlossene Kreisläufe werden analytische Lösungen angegeben. Dabei werden sowohl die Bedingungen der Beheizung bei konstanter Wärmestromdichte als auch bei konstanter Temperatur untersucht. Es stellt sich heraus, daß die viskose Dissipation und die Druckarbeit Einflüsse von entgegengesetzter Tendenz auf die Strömung im Kreislauf haben. Die viskose Dissipation kann die Strömung bei bestimmten Anordnungen der Beheizung beschleunigen, wogegen die Druckarbeit gewöhnlich überwiegt und die Strömung verzögert.

ОБ ЭФФЕКТАХ ВЯЗКОЙ ДИССИПАЦИИ И РАБОТЫ СЖАТИЯ В КОНТУРАХ СО СВОБОДНОЙ КОНВЕКЦИЕЙ

Аннотация Теоретически исследованы эффекты вязкой диссипации и работы сжатия в контурах с ламинарной свободной конвекцией. Дан вывод основных уравнений. В отличие от ранее проведенного исследования, где рассматривались только эффекты диссипации, в данной работе показано, что диссипация и работа сжатия сравнимы по величине и должны учитываться одновременно. Представлены аналитические решения для нескольких открытых и закрытых контуров. Проанализирован нагрев как при постоянном тепловом потоке, так и постоянной температуре. Установлено, что вязкая диссипация и работа сжатия оказывают противоположное влияние на течение в контуре. Диссипация может усиливать течение только при определенных условиях подвода тепла, в то время как работа сжатия всегда доминирует и замедляет течение.